I have to apologize because this is not the normal sort of question for this site, but there have been times in the past where MO was remarkably helpful and kind to undergrads with similar types of question and since it is worrying me increasingly as of late I feel that I must ask it.

My question is: what can one (such as myself) contribute to mathematics?

I find that mathematics is made by people like Gauss and Euler - while it may be possible to learn their work and understand it, nothing new is created by doing this. One can rewrite their books in modern language and notation or guide others to learn it too but I never believed this was the significant part of a mathematician work; which would be the creation of original mathematics. It seems entirely plausible that, with all the tremendously clever people working so hard on mathematics, there is nothing left for someone such as myself (who would be the first to admit they do not have any special talent in the field) to do. Perhaps my value would be to act more like cannon fodder? Since just sending in enough men in will surely break through some barrier.

Anyway I don't want to ramble too much but I really would like to find answers to this question - whether they come from experiences or peoples biographies or anywhere.

Thank you.
It's not *mathematics* that you need to contribute to. It's deeper than that: how might you contribute to humanity, and even deeper, to the well-being of the world, by pursuing mathematics? Such a question is not possible to answer in a purely intellectual way, because the effects of our actions go far beyond our understanding. We are deeply social and deeply instinctual animals, so much that our well-being depends on many things we do that are hard to explain in an intellectual way. That is why you do well to follow your heart and your passion. Bare reason is likely to lead you astray. None of us are smart and wise enough to figure it out intellectually.

The product of mathematics is clarity and understanding. Not theorems, by themselves. Is there, for example any real reason that even such famous results as Fermat's Last Theorem, or the Poincaré conjecture, really matter? Their real importance is not in their specific statements, but their role in challenging our understanding, presenting challenges that led to mathematical developments that increased our understanding.

The world does not suffer from an oversupply of clarity and understanding (to put it mildly). How and whether specific mathematics might lead to improving the world (whatever that means) is usually impossible to tease out, but mathematics collectively is extremely important.

I think of mathematics as having a large component of psychology, because of its strong dependence on human minds. Dehumanized mathematics would be more like computer code, which is very different. Mathematical ideas, even simple ideas, are often hard to transplant from mind to mind. There are many ideas in mathematics that may be hard to get, but are easy once you get them. Because of this, mathematical understanding does not expand in a monotone direction. Our understanding frequently deteriorates as well. There are several obvious mechanisms of decay. The experts in a subject retire and die, or simply move on to other subjects and forget. Mathematics is commonly explained and recorded in symbolic and concrete forms that are easy to communicate, rather than in conceptual forms that are easy to understand once communicated. Translation in the direction conceptual -> concrete and symbolic is much easier than translation in the reverse direction, and symbolic forms often replaces the conceptual forms of understanding. And mathematical conventions and taken-for-granted knowledge change, so older texts may become hard to understand.

In short, mathematics only exists in a living community of mathematicians that spreads understanding and breathes life into ideas both old and new. The real satisfaction from mathematics is in learning from others and sharing with others. All of us have clear understanding of a few things and murky concepts of many more. There is no way to run out of ideas in need of clarification. The question of who is the first person to ever set foot on some square meter of land is really secondary. Revolutionary change does matter, but revolutions are few, and they are not self-sustaining --- they depend very heavily on the community of mathematicians.
@J.M., @Harrison Brown, thanks for the comments. I try to write what seems real. By now, I have no cause to fear how I will be judged, which makes it much easier for me. It's gratifying when my reality means something to others. – Bill Thurston Oct 30 at 17:10

Please let me join the chorus of praise for Thurston's essays! A personal favorite is Thurston's wonderful Foreword to Mircea Pitati's collection "The Best Writing on Mathematics 2010". The Foreword begins "Mathematics is commonly thought to be the pursuit of universal truths" ... (Google Books will find the phrase). Then Thurston's essay goes on to argue that mathematics is much more than that. – John Sidles May 31 at 14:46

You don't have to be Michael Jordan to play basketball.

I had the privilege of discussing similar concerns with regard to theoretical physics with the late Richard Feynman. He told me the following, which has always served me in good stead: "You keep on learning and learning, and pretty soon you learn something no one has learned before." That was his "advice"; my advice? Go for it!

Hironaka said much the same to me in my first year in grad school. It was something like, "You may think you'll never prove a theorem, but if you study a subject long enough, soon you'll see that you are proving something new in it." – Lubin May 31 at 3:35
One can rewrite their books in modern language and notation or guide others to learn it too but I never believed this was the significant part of a mathematician work; which would be the creation of original mathematics is a widely-held fallacy. Perhaps 90% of the work of mathematicians is not brilliantly original creation, but the challenge of digesting and reworking over the cumulative insights and re-presenting them in a useful way for others (this generation and the next). This is a huge task, and it’s often a very satisfyingly creative task which will well reward one with new insights, understanding of consequences of the major insights, and understanding their underpinnings.

And part of this is: teach and write well. Good exposition is perhaps the most important contribution that “lesser mortals” can make (and perhaps our collective livelihood depends on it -- Rota has some perceptive things to say about that in his Indiscrete Thoughts). You can get started on this even as an undergraduate: once you have understood something, share it with others. Give a good talk in your local math club. Pay it forward!

While I certainly agree that exposition and teaching are extremely important, I strongly disagree with the notion that 90% of the work of mathematicians is “the challenge of digesting and reworking over the cumulative insights and re-presenting them in a useful way for others”. We prove theorems (new and original ones) too! I think it is important for young people to realize that math is so huge that even if you are pretty ordinary, there is plenty of room to have original insights and do good work. – Andy Putman

Well, Erdős wrote a lot of things. – Todd Trimble

"Does a theorem make a sound if nobody can understand it?" – Greg Graviton

Terry Tao wrote on this subject; I think you will be happy with his conclusions:
http://terrytao.wordpress.com/career-advice/does-one-have-to-be-a-genius-to-do-maths/

An excerpt: "The number of interesting mathematical research areas and problems to work on is vast – far more than can be covered in detail just by the “best” mathematicians, and sometimes the set of tools or ideas that you have will find something that other good mathematicians have overlooked, especially given that even the greatest mathematicians still have weaknesses in some aspects of mathematical research. As long as you have education, interest, and a reasonable amount of talent, there will be some part of mathematics where you can make a solid and useful contribution."
What's a mathematician to do? – MathOverflow

Beren Sanders

+1; Terence Tao's career advice is always worth a read. – Qiaochu Yuan Oct 26 at 18:59

Also worth quoting from the same article: "It's also good to remember that **professional mathematics is not a sport***" – Thierry Zell Oct 26 at 19:29

That article by Terrence Tao is a wonderful read. But particularly the sentence that Thierry Zell is quoting seems a bit too optimistic to me. Until one becomes a tenured professor, mathematics is exactly the sort of sport that Prof. Tao doesn't want it to be. That's just the brutal truth of any kind of competition for limited places. So while we like to think of ourselves as idealists, who do research with the sole aim of advancing knowledge, in practice many people are forced to spend time on "routine" maths to get enough publications, rather than on the things they find most interesting. – Alex Bartel Oct 27 at 0:54

@Alex: good point, sad as I am to admit it. – Thierry Zell Oct 30 at 4:02

I'm not really qualified to answer this...or perhaps I am, since I'm no Terry Tao! I think what we bring to mathematics is the unique perspective afforded by our own individual experience. The more of us who are trying to do mathematics, the better, because this increases the diversity of perspectives in approaching the myriad problems we face. Also, as is popular to say, there is far more mathematics to do than any handful of powerful mathematicians can do. This is a "big tent".

Another thing that I've read someplace...someone was describing the difference between Kolmogorov and Israel Gelfand, and this person wrote/said the following: "when Kolmogorov went into a new mathematical landscape he immediately looked for the tallest mountain and climbed it, when Gelfand entered the same landscape, he immediately began building roads." (Someone please fill mud in on the proper location of this quote...I think it was the Notices...)

Others have said that most of what we do as mathematicians is organize and clean things up to clear the way for a future polymath, like a von Neumann, to really make some progress, and some road builders have built some unbelievable roads. (Didn't Serre say something like he spent most of his career rewriting other people's work?)

I don't know about you, but I'm fine with this! It's much better than having my name on the location of a transistor on some unknown circuitboard on the space shuttle...not that there's anything wrong with that!

Anyhow, don't worry about having things to do...there are plenty!

Let me add two quotations:

1. Fermat's motto was "Multi pertransibunt et augebitur scientia" (many will pass through and knowledge will be increased). At another occasion he wrote about "passing the torch to the next generation",
which I find particularly nice.

2. "When kings are building, carters have work to do". Kronecker quoted this, in his letter to Cantor of September 1891.

One possible way to think about this midnight question might be to ask what smaller results have been important or interesting to you, and then to appreciate the existence of the mathematicians who have discovered/invented it. My guess would be that everyone can compile a list of such results by players who are not in the league of Gauss and Euler. My own list would be quite long, and some of the results are recent enough to have been discovered by users of MO.

If you are an undergraduate, you don't yet know what you can contribute to mathematics, and in particular you don't yet know whether, or at what level, you can create original mathematics. Fortunately, you don't have to work this out entirely for yourself; if you go on to graduate work in mathematics, you will have an advisor, whose job is to help you get the most out of your potential. Even after a student finishes a PhD, she's not alone; much of the best mathematical work today is collaborative, and whatever weaknesses you may think you have can be compensated for by the strengths of your collaborators (while you compensate for their weaknesses with your strengths).

It ain't easy, but it can be done.

I don't agree that an undergraduate can't create original mathematics, if that's what you mean to say. Maybe they require more help from an advisor than graduate students typically do, but that's no reason to discourage the OP or any undergraduate from trying to do new mathematics as soon as possible. – Justin Campbell Oct 27 at 16:49

I don't think you should discourage them, but I also think that it's extremely unhealthy for undergraduates to feel pressure to "do research". Most undergraduates (including most undergraduates who go on to top grad schools and end up becoming top researchers) don't prove any original theorems during their undergraduate years. – Andy Putman Oct 27 at 16:56

Agreed, Andy: but "pressure" is not quite the sentiment I meant to express. Isn't it healthy for undergraduates who want to go into research to at least have some exposure to the practices of working mathematicians, even if they don't have actual research experience? That way, during the crucial period when they are first expected to produce original mathematics, the process doesn't seem so bewildering. This is more for the purpose of creating interesting mathematical narratives than publishing paper after paper, of course. – Justin Campbell Oct 27 at 19:29

Walking before running, and all that – Yemon Choi Oct 27 at 20:39
I didn't mean to say that an undergraduate can't create original mathematics, only that no undergrad should feel she'll never create original mathematics just because, as an undergrad, she can't see how she's ever going to be able to do it. – **Gerry Myerson** Oct 28 at 0:47

I like the cannon fodder analogy -- made me laugh! I think of myself that way sometimes. Also, though, deep down I like to think I might be smarter than I realize. If I keep at it, maybe I will tap into some hidden reserves of insight. Also, remember the line from Ecclesiastes:

"I returned, and saw under the sun, that the race is not to the swift, nor the battle to the strong, neither yet bread to the wise, nor yet riches to men of understanding, nor yet favour to men of skill, nor yet theorems to mathematicians of brilliance; but time and chance happeneth to them all."

OK maybe the part about theorems is not in the original, but you get my point :-)

I love Orwell's translation of that into "modern English" from _Politics and the English Language_: "Objective considerations of contemporary phenomena compel the conclusion that success or failure in competitive activities exhibits no tendency to be commensurate with innate capacity, but that a considerable element of the unpredictable must invariably be taken into account." – **Harry Gindi** Nov 14 at 5:51

@Harry: Oh my how ugly. – **timur** Aug 9 at 2:47

I suspect that most interesting mathematical results raise more questions than they settle, so in this case you would not have anything to worry about.

Muad you raised not only a very important question but also a fairly enigmatic one. I completed my Maters in Mathematics 13 years ago and left mathematics then and there, what I realized during these years was that I could not sleep without thinking something new to discover or invent in pure mathematics. The numbers kept on haunting me. I was fascinated by Relativistic Mathematics as well had a great passion for Number Theory. I tried to find a suitable topic to do some original research work but left that to enter a totally different field. I opened a restaurant named e=mc^2. But still couldn't sleep well. So after running a successful talk-of-the-town restaurant for almost 4 years, I decided to turn to Computer Graphics, another of my passions as I was very involved in fine arts and poetry as well. Having graduated from Vancouver Film School, I am now a Computer Graphics Technical Director at a Visual Effects Studio, where I deal with mathematics in a more intimate way on a day to day basis. The point of all this autobiographical ramblings of mine is this : If you have the right motivation, and if you are curious enough, you have something to discover. My nights are still sleepless as I am yet to find something to discover !!! So I think mathematics is inside you waiting for it to be discovered, just be passionate enough !!
@Alok: Note that I didn't ask the question, but was just the last person to edit it. (I agree that the display does not make this very clear for community wiki questions, and I sympathize with your confusion. The same misreading has been done by many, many other MO users, including me.) – Pete L. Clark Oct 30 at 16:48

@Pete: Thanks for the information. I see that the questions is asked by maud. I will be careful to look up at the user posing the question before answering. – Alok Gandhi Oct 30 at 18:01

On this issue I find a deal of comfort in the concluding paragraph of G. H. Hardy's *A Mathematician's Apology*:

The case for my life, then, or for that of any one else who has been a mathematician in the same sense which I have been one, is this: that I have added something to knowledge, and helped others to add more; and that these somethings have a value which differs in degree only, and not in kind, from that of the creations of the great mathematicians, or of any of the other artists, great or small, who have left some kind of memorial behind them.

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Hardy was pretty clearly clinically depressed when he wrote that book. From a purely objective standpoint, it would have been much more reasonable for him to have written something like "I was arguably the best analytic number theorist, and the greatest British mathematician, of the first half of the twentieth century, and my reputation will be steadily on the rise more than 50 years after my death. That's pretty good, right?" – Pete L. Clark Oct 27 at 7:44

Not really happy with that comment. The merciless quality of the writing there traces back to the Bloomsbury group (Hardy was on the edge of that clique). While we know he did suffer depression, it is a disabling condition by the time doctors should be involved, and not then one that allows for producing literary classics – Charles Matthews Oct 27 at 11:22

Is there any source for this diagnosis of depression other than C. P. Snow's foreword to AMA? – Yemon Choi Oct 27 at 20:40

@Charles and Yemon: it was just a comment, and I am not a historian or a psychologist. Please add "I think" at the beginning of it. But here is what Rebecca Goldstein said in an interview available online: "He wrote the book after his first suicide attempt and before his second—and successful—suicide attempt." I also find, as I said more lightly above, an extreme under-estimate of his own worth and abilities. Maybe there are also other reasons for that, as Charles suggests. Anyway, to be sure, I can't "diagnose" anyone of anything; I can only state my opinion. – Pete L. Clark Oct 30 at 8:39

Mathematics needs people to create, to explain, to synthesize, to apply, to teach, to learn, even to proselytize (in socially acceptable ways). If you want fame bordering on immortality, solve a very hard problem or create something that both solves and poses hard problems. If you want instead to be a great contributor to mathematics, do as much of the above as your heart and mind allow.

What would you like to do for mathematics?

Gerhard "Ask Me About System Design" Paseman, 2010.10.27
@muad: You say you are not strong in mathematics. I don't know whether this is really such. But let's assume that you cannot do math discoveries for the purpose of this discussion.

You may do something other.

You may consider to write software for mathematics. One such project is TeXmacs. I would be grateful if you would manage to learn Scheme programming language and program proper creation of LaTeX macroses when exporting from TeXmacs to LaTeX.

Do you like my idea of what you can do?

I consider to take time to do this myself, but this would steal from my valuable time to do mathematics.

7  This is a nice and helpful answer. -- Gil Kalai Dec 19 at 21:08