PROBLEM 3: COLOURED CIRCLES



ITYM 2015

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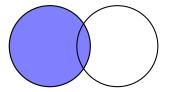
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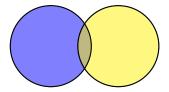


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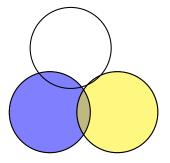
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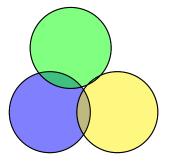
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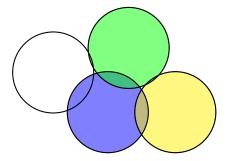
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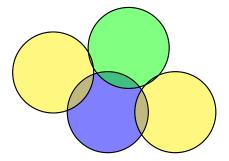
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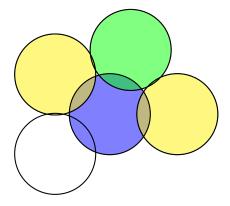
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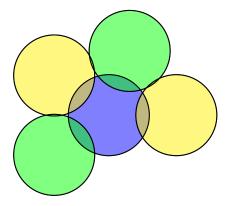
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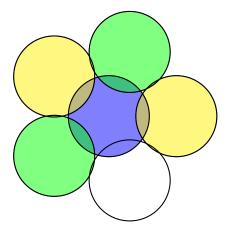
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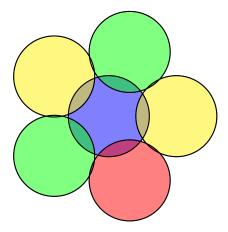
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Notations

For all lower bound, we suppose that *n* is big enough.

- $n \rightarrow \#$ balls
- $d \rightarrow \text{dimension}$
- $k \rightarrow$ each point of the plane is covered by at most k balls
- $\mathcal{C}_d(n,k) \rightarrow \text{ minimal } \# \text{ of colors needed to colour the}$ balls, when Alice plays as she wishes $C_d^c(n,k) \rightarrow$ all balls have the same radius

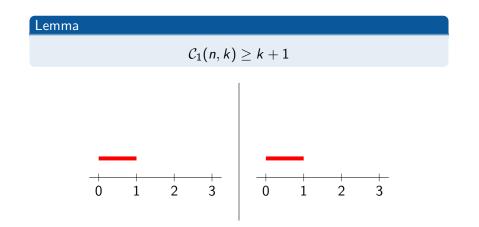
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 $\mathcal{C}_1^c(n,k) \leq 2k-1$

$k-1\{$ \blacksquare $\}k-1$

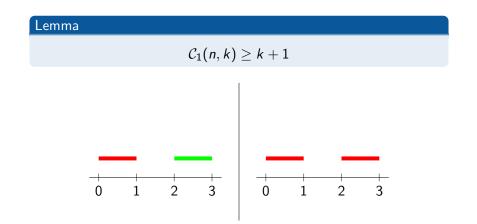
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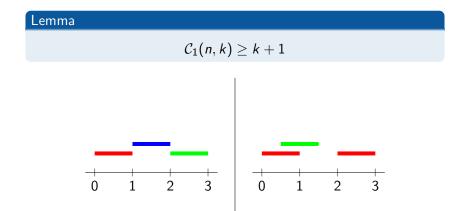


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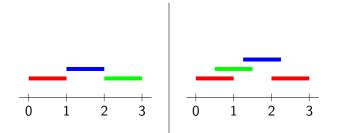


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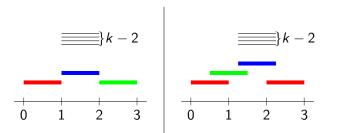


$C_1(n,k) \ge k+1$



d=1	
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$C_1(n,k) \ge k+1$



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$$\mathcal{C}_1(n,k) \geq \frac{5k-1}{4}$$

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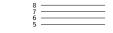
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$$\mathcal{C}_1(n,k) \geq \frac{5k-1}{4}$$

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$$\mathcal{C}_1(n,k)\geq \frac{5k-1}{4}$$



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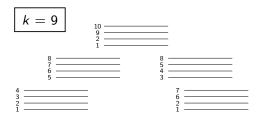
$$\mathcal{C}_1(n,k)\geq \frac{5k-1}{4}$$



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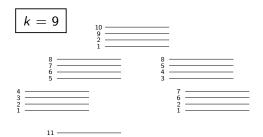
$$\mathcal{C}_1(n,k)\geq \frac{5k-1}{4}$$



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$$\mathcal{C}_1(n,k)\geq \frac{5k-1}{4}$$

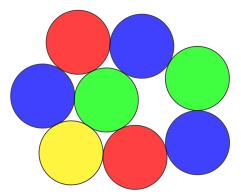


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	$\begin{array}{l} d = 2 \\ \bullet \circ \circ \circ \circ \circ \end{array}$	$d \in \mathbf{N}$ 0000

 $\mathcal{C}_2(n,2) \geq 4$



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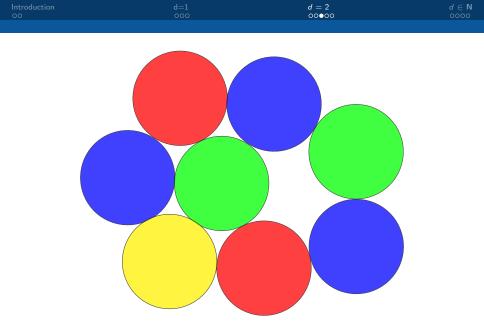
Four-colors theorem

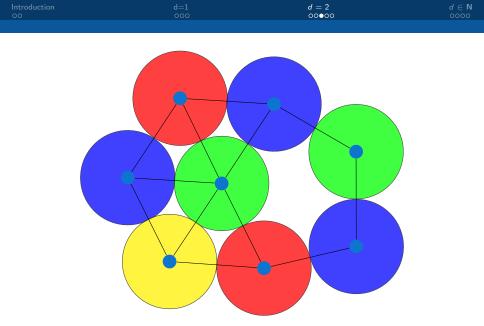
The vertices of every planar graph can be colored with at most four colors so that no two adjacent vertices receive the same color.

$$\left\{ egin{array}{l} \mathcal{C}_2(n,2)\geq 4 \ \mathcal{C}_2(n,2)\leq 4 \end{array}
ight. \Rightarrow \mathcal{C}_2(n,2)=4^*$$

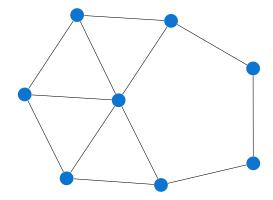
*Supposing that Alice places all her circles, and then Bob colours them = -9 and = -9

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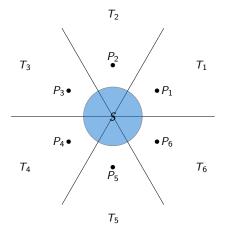
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Theorem

One disc intersects at most 7k - 1 other discs. (Generalization of shortlist IMO 2003)

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Theorem

One disc intersects at most 7k - 1 other discs. (Generalization of shortlist IMO 2003)

Corollary

$\mathcal{C}_2^c(n,k) \leq 7k$

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Propostion

$$\mathcal{C}_2(n,k)\geq \frac{7k}{3}$$

A *complete graph* is a graph in which every pair of distinct vertices is connected by a unique edge.

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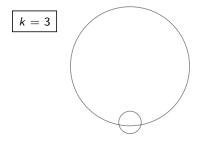
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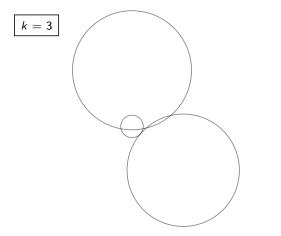


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Problem 3: Coloured Circles

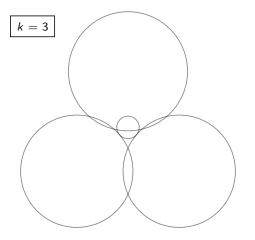
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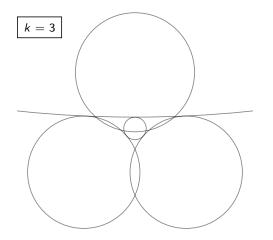
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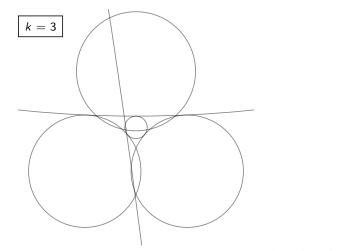


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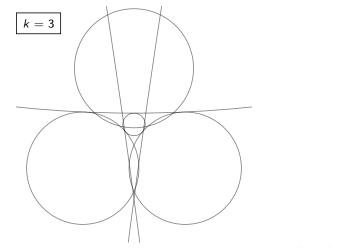


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	$d \in N$
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$\mathcal{C}_d(n,k) \geq k+d$

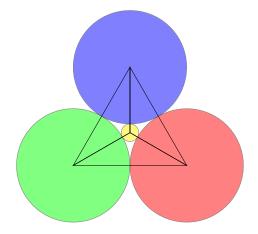
Lemma

$$\mathcal{C}_d(n,k) \geq \frac{k}{2}(d+2)$$

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A *simplex* is a generalization of the notion of a triangle or tetrahedron to arbitrary dimensions

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$\mathcal{C}_d^c(n,k) \leq 3^d k$

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Problem 3: Coloured Circles

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$$\lim_{d\to+\infty}\mathcal{C}_d(n,k)=n$$

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Summary

$$\begin{array}{rcl} \mathcal{C}_{1}^{C}(n,k) &\leq 2k-1 \ \mathcal{C}_{2}^{C}(n,k) &\leq 7k \ \mathcal{C}_{d}^{C}(n,k) &\leq 3^{d}k \end{array}$$

$$egin{array}{rcl} k+1&\leq&\mathcal{C}_1(n,k)\ k+2&\leq&\mathcal{C}_2(n,k)\ k+d&\leq&\mathcal{C}_d(n,k) \end{array}$$

$$\begin{array}{rcl} \frac{5k-1}{4} & \leq & \mathcal{C}_1(n,k) \\ \frac{7k}{3} & \leq & \mathcal{C}_2(n,k) \\ \frac{k}{2}(d+2) & \leq & \mathcal{C}_d(n,k) \end{array}$$

Thank you for your attention 🙂 !